Problem 7.12

(a) $H(z) = (1-z^{-1})(1+z^{-2})(1+z^{-1})$ MULTIPLY OUTER FACTORS

$= (1-z^{-2})(1+z^{-2}) = 1-z^{-4}$

$\therefore$ $y[n] = x[n] - x[n-4]$

(b) $H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = 1 - e^{-j4\hat{\omega}}$

(c) $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}(e^{+j2\hat{\omega}} - e^{-j2\hat{\omega}})$

$= 2j\, e^{-j2\hat{\omega}} \sin 2\hat{\omega} = (2\sin 2\hat{\omega})\, e^{j(\pi/2 - 2\hat{\omega})}$

MAG: $2\sin 2\hat{\omega}$ ALTHOUGH THIS HAS A SIGN CHANGE FOR $\hat{\omega} < 0$

PHASE: $\pi/2 - 2\hat{\omega}$

(d) BLOCK WHEN $H(e^{j\hat{\omega}}) = 0$

$\therefore$ SOLVE $2\sin 2\hat{\omega} = 0$

$\Rightarrow \hat{\omega} = 0, \pi/2, \pi, -\pi/2$

(e) Need $H(e^{j\pi/3})$ because that is the frequency of the input.

$H(e^{j\pi/3}) = \left(2\sin\frac{2\pi}{3}\right) e^{j(\pi/2 - 2\pi/3)}$

$= 2\left(\frac{\sqrt{3}}{2}\right) e^{j(3\pi/6 - 4\pi/6)}$

$= \sqrt{3}e^{-j\pi/6} = \sqrt{3}e^{j\pi}e^{-j\pi/6} = \sqrt{3}\, e^{j5\pi/6}$

$\therefore$ OUTPUT IS: $y[n] = \sqrt{3}\cos\left(\frac{\pi n}{3} + \frac{5\pi}{6}\right)$

Problem 8.3

$$y[n] = \frac{1}{2}y[n-1] + \frac{1}{3}y[n-2] - x[n] + 3x[n-1] - 2x[n-2]$$

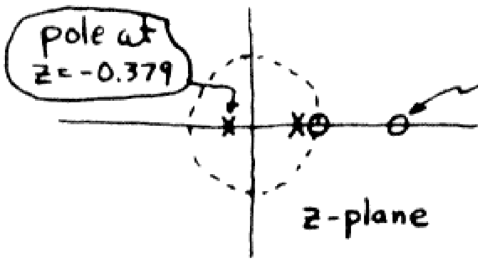
$$Y(z) = \frac{1}{2}z^{-1}Y(z) + \frac{1}{3}z^{-2}Y(z) - X(z) + 3z^{-1}X(z) - 2z^{-2}X(z)$$

$$\left(1 - \frac{1}{2}z^{-1} - \frac{1}{3}z^{-2}\right)Y(z) = (-1 + 3z^{-1} - 2z^{-2})X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-1 + 3z^{-1} - 2z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{1}{3}z^{-2}}$$

change to positive powers of z to find roots.

$$H(z) = -\frac{z^2 - 3z + 2}{z^2 - \frac{1}{2}z - \frac{1}{3}} = -\frac{(z-2)(z-1)}{(z-0.879)(z+0.379)}$$



For the second system only the signs on $y[n-2]$ and $x[n-2]$ change, so we can write $H(z)$ immediately:

$$H(z) = -\frac{1 - 3z^{-1} - 2z^{-2}}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}} = -\frac{z^2 - 3z - 2}{z^2 - \frac{1}{2}z + \frac{1}{3}}$$

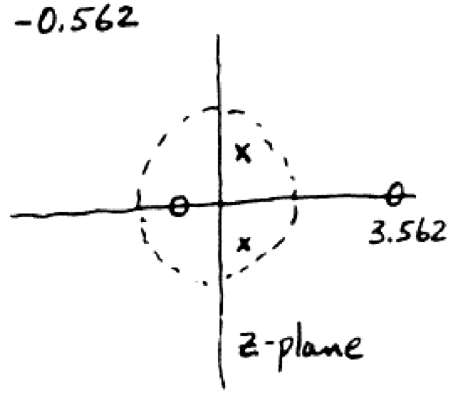
ZEROS: $\frac{3 \pm \sqrt{9+8}}{2} = 3.562, -0.562$

POLES: $0.25 \pm j0.52$

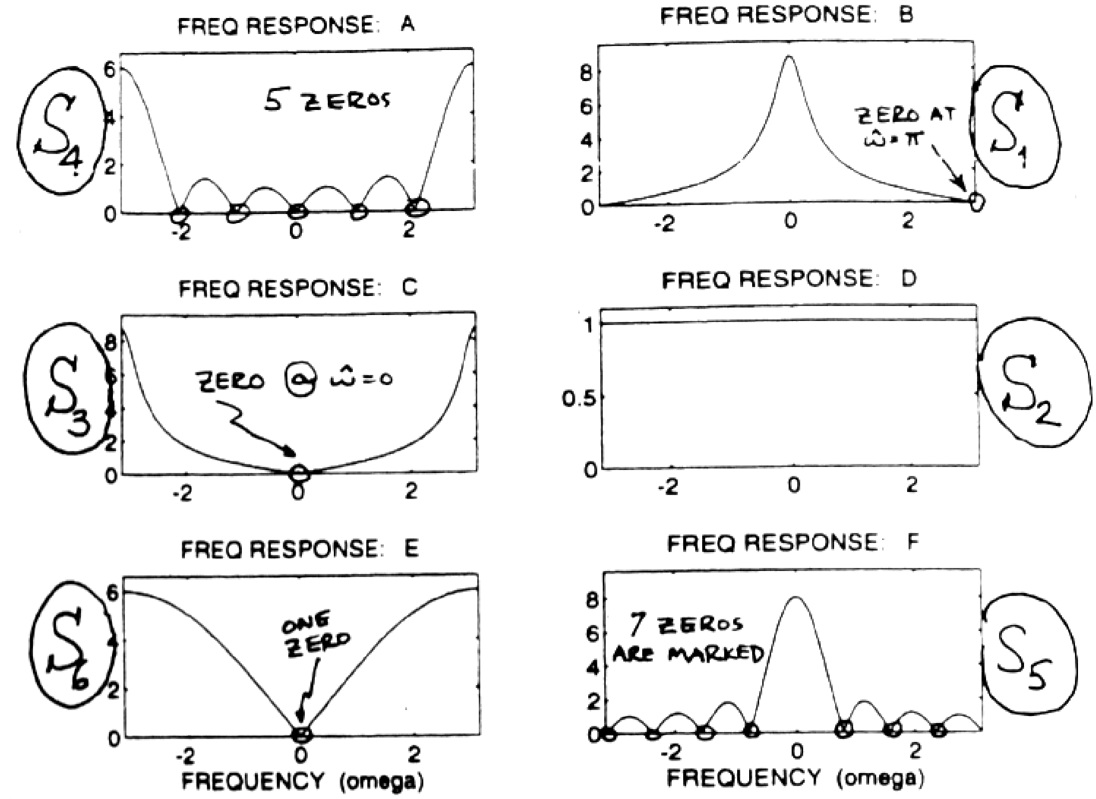
$= 0.5774e^{\pm j0.357\pi}$

ANGLE is $\pm 64.34°$

or ± 1.123 rads.



Problem 8.14



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $\mathcal{H}(z)$ or a difference equation) matches the frequency response.

S_1 : $y[n] = 0.77y[n-1] + x[n] + x[n-1]$ — LOWPASS w/ ZERO AT $\hat{\omega} = \pi$

S_2 : $y[n] = 0.77y[n-1] + 0.77x[n] - x[n-1]$ → $H(z) = \dfrac{0.77 - z^{-1}}{1 - 0.77z^{-1}}$ is ALL-PASS

S_3 : $\mathcal{H}(z) = \dfrac{1 - z^{-1}}{1 + 0.77z^{-1}}$ — HIGH-PASS w/ ZERO AT $\hat{\omega} = 0$

S_4 : $\mathcal{H}(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5}$ — "Dirichlet" HIGH PASS w/ 5 ZEROS ON U.C.

S_5 : $y[n] = \sum_{k=0}^{7} x[n-k]$ — "Dirichlet" LOW PASS w/ 7 ZEROS ON U.C.

S_6 : $\mathcal{H}(z) = 3 - 3z^{-1}$ HIGH-PASS w/ 1 ZERO

S_7 : $y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5]$

→ "Dirichlet" LOWPASS w/ 5 ZEROS ON U.C.

Problem 8.18

Multiply out $H(z)$

$$H(z) = \frac{(1-z^{-1})(1-jz^{-1})(1+jz^{-1})}{(1-0.9e^{j2\pi/3}z^{-1})(1-0.9e^{-j2\pi/3}z^{-1})}$$

$$= \frac{(1-z^{-1})(1+z^{-2})}{1-2(0.9)\cos(2\pi/3)z^{-1}+(0.9)^2z^{-2}}$$

$$= \frac{1-z^{-1}+z^{-2}-z^{-3}}{1-0.9z^{-1}+0.81z^{-2}}$$

(a) use the numerator & denominator polynomial coefficients as filter coefficients:

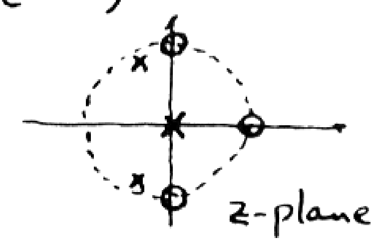
$$y[n] = 0.9y[n-1]-0.81y[n-2]+x[n]-x[n-1]+x[n-2]-x[n-3]$$

(b) Multiply numerator & denominator by z^3:

$$H(z) = \frac{(z-1)(z-j)(z+j)}{z(z-0.9e^{j2\pi/3})(z-0.9e^{-j2\pi/3})}$$

Zeroes: $z=1$, j and $-j$

Poles: $z=0$, $z=0.9e^{\pm j2\pi/3}$



(c) The zeros of the numerator polynomial are on the unit circle at $z=e^{j0}$, $z=e^{j\pi/2}$ and $z=e^{-j\pi/2}$

When $x[n] = Ae^{j\varphi}e^{j\hat{\omega}n}$, the output $y[n]$ is

$$y[n] = H(e^{j\hat{\omega}})\cdot Ae^{j\varphi}e^{j\hat{\omega}n}$$

There the output will be zero when $H(e^{j\hat{\omega}})=0$.

That is, for $\hat{\omega}=0$, $\hat{\omega}=\pi/2$ and $\hat{\omega}=-\pi/2$.