The Decibel Scale

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It is often convenient to compare two quantities in an audio system using a proportionality ratio. For example, if a linear amplifier produces 2 volts (V) output amplitude when its input amplitude is 100 millivolts (mV), the *voltage gain* is expressed as the ratio of output/input: 2V/100mV = 20. As long as the two quantities being compared have the same units--*volts* in this case--the proportionality ratio is dimensionless.

If the proportionality ratios of interest end up being very large or very small, such as 2×10^5 and 2.5×10^{-4}, manipulating and interpreting the results can become rather unwieldy. In this situation it can be helpful to compress the numerical range by taking the *logarithm* of the ratio. It is customary to use a base-10 logarithm for this purpose. For example,

$$\log_{10}(2 \times 10^5) = 5 + \log_{10}(2) = 5.301$$

and

$$\log_{10}(2.5 \times 10^{-4}) = -4 + \log_{10}(2.5) = -3.602$$

If the quantities in the proportionality ratio both have the units of *power* (e.g., watts), or *intensity* (watts/m^2), then the base-10 logarithm $\log_{10}(\text{power}_1/\text{power}_0)$ is expressed with the unit *bel* (symbol: *B*), in honor of Alexander Graham Bell (1847 -1922).

The *decibel* is a unit representing one tenth (deci-) of a bel. Therefore, a figure reported in decibels is ten times the value reported in bels. The expression for a proportionality ratio expressed in decibel units (symbol dB) is:

$$dB \equiv 10 \cdot log_{10}\left(\frac{power_1}{power_0}\right)$$

Common Usage

The power dissipated in a resistance *R* ohms can be expressed as V^2/R, where *V* is the voltage across the resistor. If we compare two power levels specified with the *same* resistance R, we can express the dB ratio as

$$10 \log_{10}((V_2^2/R)/(V_1^2/R)) = 10 \log_{10}((V_2/V_1)^2) = 20 \log_{10}(V_2/V_1)$$

Also, for a plane or spherical wave the acoustic intensity is proportional to the square of the acoustic pressure, leading again to an expression

$$10 \log_{10}((P_2^2/\rho_0 c)/(P_1^2/\rho_0 c)) = 20 \log_{10}(P_2/P_1) ,$$

where $\rho_0 c$ is the specific acoustic impedance.

These results have led to the *casual* definition of dB simply as 20 $\log_{10}(\ )$ of the ratio of any two voltages or two pressures, *even when the electrical or acoustical impedance, respectively, are not actually the same*. While this is not technically correct, it has become pretty common in the field of engineering.

In many cases it is most common to use an explicit reference level in the denominator of the ratio, and then to specify the reference explicitly.

<u>Sound Pressure Level (SPL)</u>:

- $L_P = 20 \log_{10}(P/P_{ref})$, expressed as dB re P_{ref}

The reference pressure P_{ref} for most hearing-related measurements in air is 20 μPa (20 micropascal, or $2\text{x}10^{-5}$ pascal), although sometimes SPL measurements are expressed relative to 1 μbar (1 μbar = 0.1 Pa). A sound with effective pressure of 20 μPa is roughly the threshold of hearing for average listeners in the 1 kHz frequency range.

Expressing sound levels in decibels is also appropriate because human loudness perception tends to be logarithmic: a 10dB increase in SPL for a pure tone corresponds roughly to a doubling of perceived loudness.

<u>Voltage</u>:

$20 \log_{10}(V/V_{ref})$, expressed as dB*x*, such as

- dBV when V_{ref} = 1 V RMS (root-mean-square)
- dBu (or dBv) when V_{ref} = 0.775 V RMS

Note that 0.775 volts is the voltage necessary to dissipate 1mW (milliwatt) in a 600 ohm load, a common standard in the telecommunications industry.

<u>Power</u>:

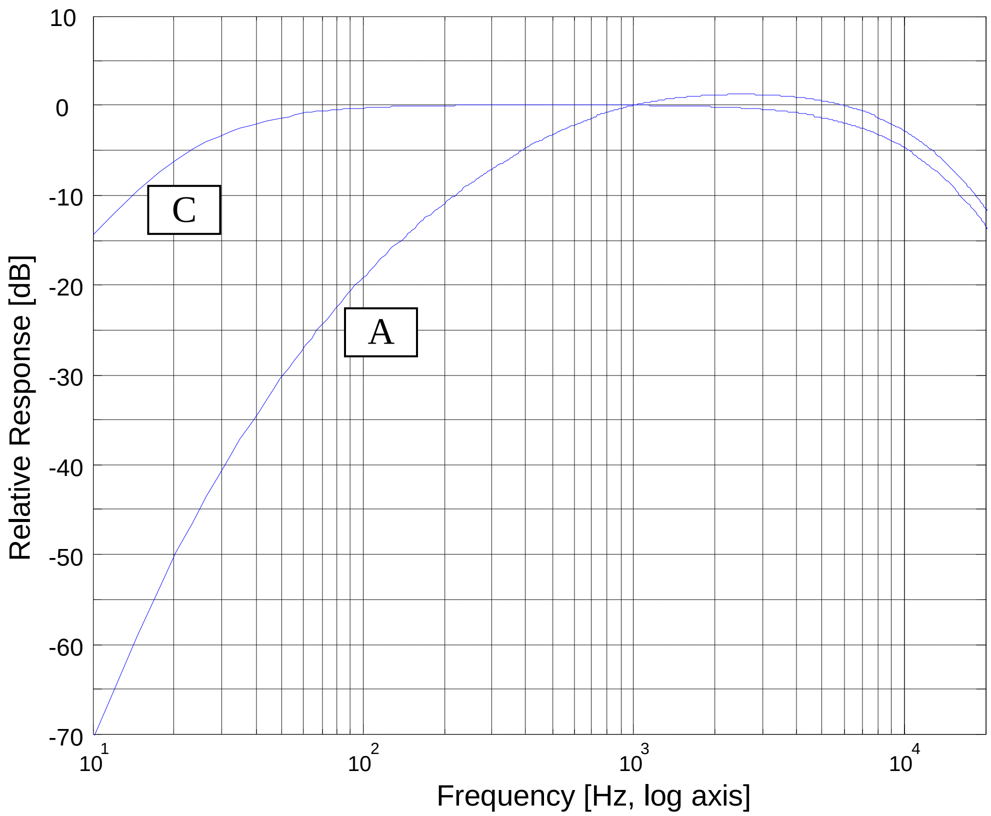
$10\log_{10}(P/P_{ref})$, expressed as dB*x*, such as

- dBm when P_{ref} = 1 mW
- dBW when P_{ref} = 1 W

Frequency Weighting

dB measurements may be made using a frequency weighting filter, sometimes called a weighting *network*, to make the measurement depend on the frequency distribution of the signal's spectrum. The input signal is passed through the weighting filter prior to the signal level determination.

Several weighting filters have been standardized. Two standard weighting filters, arbitrarily designated A and C, are the most common. The A-weighting filter corresponds roughly to the average sensitivity of the human ear at low to moderate sound levels, while the C-weighting filter approximates the ear's sensitivity at high sound levels. Note that the A-weighting filter tends to emphasize spectral components in the 2 to 5 kHz range while reducing the contribution from lower and higher frequencies. The C-weighting filter has a flatter response over much of the audio frequency band.



Weighting filter response: C-weighting and A-weighting

It is important to understand the influence of the weighting network because noise levels and signal levels in audio systems are sometimes specified in this manner.

References

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